2001/3/A/4.

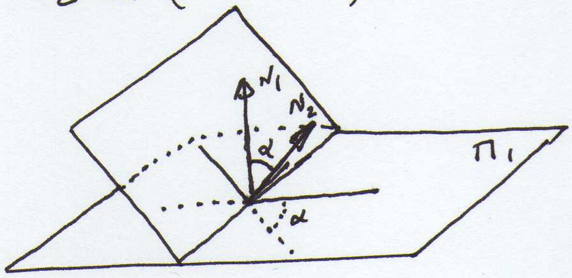
$\Pi_1 \equiv 2x + 5 = 0$

$\Pi_2 \equiv 3x + 3y - 4 = 0.$

a) Ángulo entre los planos

$\vec{N_1} = (2,0,0) \sim \vec{N_1}\ (1,0,0) \to |\vec{N_1}| = 1$

$\vec{N_2} \equiv (3,3,0) \sim \vec{N_2}\ (1,1,0) \to |\vec{N_2}| = \sqrt{1^2+1^2} = \sqrt{2}$



$$|\vec{N_1}\cdot\vec{N_2}| = |\vec{N_1}|\cdot|\vec{N_2}|\cdot\cos\alpha$$

$$|\vec{N_1}\cdot\vec{N_2}| = |1\cdot 1 + 0\cdot 1 + 0\cdot 0| = 1$$

$$\cos\alpha = \frac{|\vec{N_1}\cdot\vec{N_2}|}{|\vec{N_1}|\,|\vec{N_2}|} = \frac{1}{1\cdot\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\boxed{\alpha = 45^\circ}$$

b) Plano Π que pasa por $O(0,0,0)$ y es perp. a los dos

$$\text{Para que } \Pi \left\{ \begin{array}{l} \perp \Pi_1 \Rightarrow \vec{N_\pi} \perp \vec{N_1} \\ \perp \Pi_2 \Rightarrow \vec{N_\pi} \perp \vec{N_2} \end{array} \right\} \vec{N_\pi} = \vec{N_1} \times \vec{N_2} \Rightarrow$$

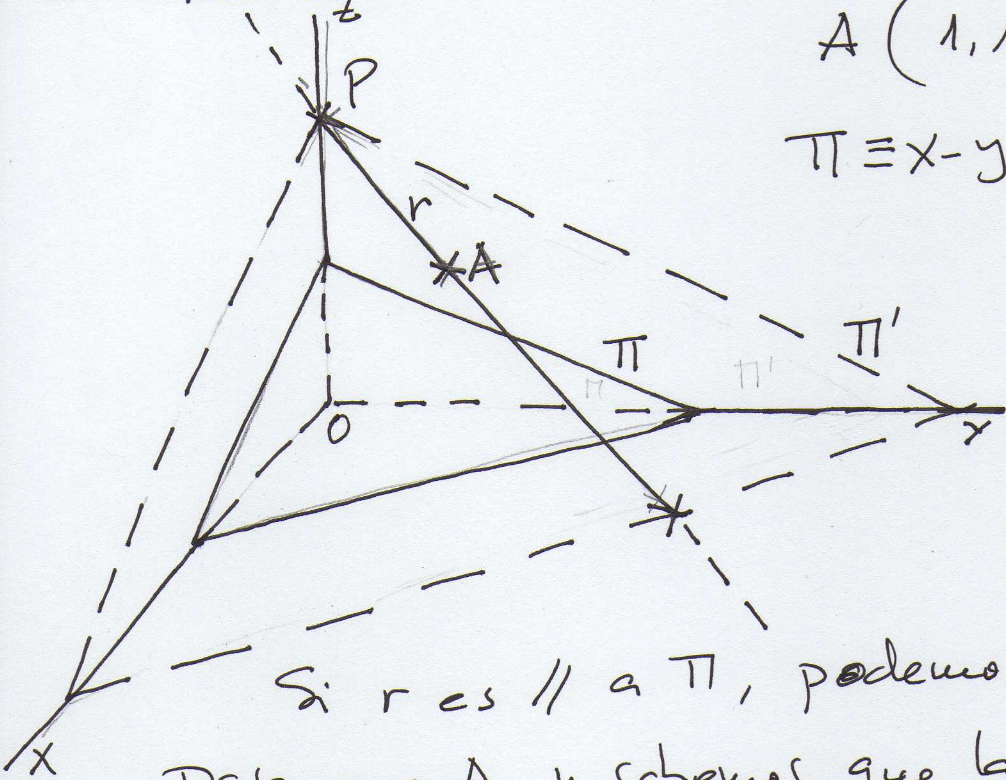
$$\vec{N_\pi} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{vmatrix} = \vec{k} = (0,0,1)$$

$$\Pi \equiv 0x + 0y + 1z + D = 0$$

como pasa por $O(0,0,0) \Rightarrow D = 0 \Rightarrow \boxed{\Pi \equiv z = 0}$

Π es OXY.

2009/5/A/4



$A(1,1,-1)$

$\pi \equiv x - y + z = 1$

$r \begin{cases} \text{Pasa por } A \\ \text{Corta a } OZ \\ \parallel \text{ a } \pi. \end{cases}$

Si r es $\parallel$ a π, podemos hacer $\pi' \parallel \pi$ que pase por A y sabemos que la contendrá:

$\pi' \equiv x - y + z + D = 0$

$A \in \pi' \Rightarrow 1 - 1 + (-1) + D = 0 \quad D = 1 \Rightarrow \boxed{\pi' \equiv x - y + z + 1 = 0}$

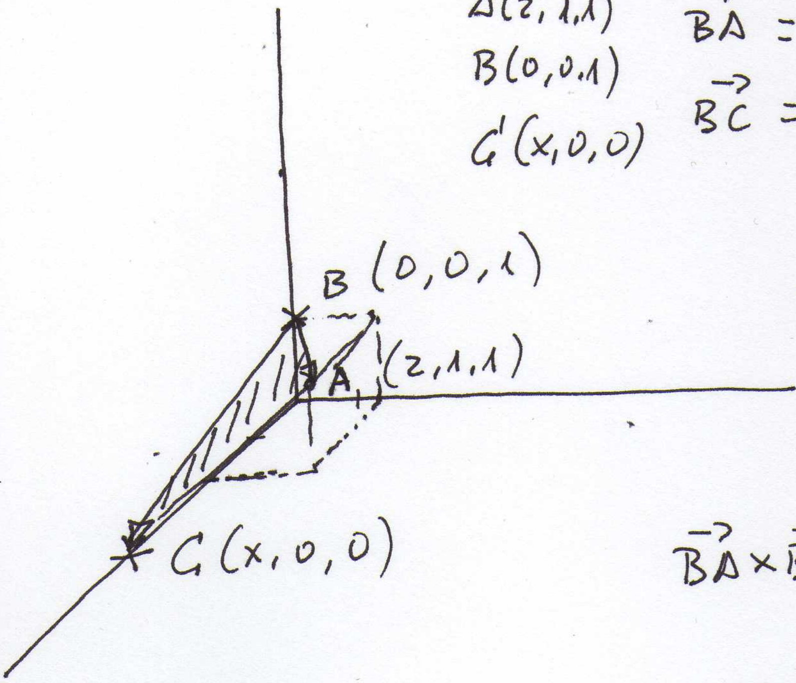
π' corta a OZ en: $\left.\begin{matrix} x = 0 \\ y = 0 \end{matrix}\right\}$ $\pi' \equiv 0 - 0 + z + 1 = 0;\ z = -1$

$P(0,0,-1)$

Por tanto, conocidos $A(1,1,-1)$ y $P(0,0,-1)$ podemos calcular $r \equiv \left\{\begin{matrix} \vec{PA} = (1,1,0) \\ \dot{P}(0,0,-1) \end{matrix}\right\} \Rightarrow \boxed{r \equiv \frac{x}{1} = \frac{y}{1} = \frac{z+1}{0}}$

$\Rightarrow r \equiv \begin{cases} x = y \\ 0 \cdot y = z + 1; \end{cases} \longrightarrow \boxed{\left.\begin{matrix} x - y = 0 \\ z = -1 \end{matrix}\right\} \equiv r}$

2008/3/B/4.



$A(2,1,1)$ $\quad \vec{BA} = (2,1,0)$

$B(0,0,1)$

$C(x,0,0)$ $\quad \vec{BC} = (x,0,-1)$

$$S = \frac{1}{2}|\vec{BA}\times\vec{BC}| = 2$$

$$\vec{BA}\times\vec{BC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 1 & 0 \\ x & 0 & -1 \end{vmatrix} =$$

$$= -\vec{i} - x\vec{k} + 2\vec{j} = (-1, 2, -x)$$

$$S = \frac{1}{2}|\vec{BA}\times\vec{BC}| = 2; \quad \boxed{|\vec{BA}\times\vec{BC}| = 4}$$

$$|\vec{BA}\times\vec{BC}| = \sqrt{(-1)^2+2^2+(-x)^2} = 4; \quad \sqrt{5+x^2} = 4; \quad 5+x^2 = 16;$$

$$x^2 = 11; \quad x = \pm\sqrt{11}$$

Hay dos sol:

$$\boxed{C_1 = (\sqrt{11}, 0, 0) \quad y \quad C_2 = (-\sqrt{11}, 0, 0)}$$